Art. XIII.—*Results of Observations with the Kater's Invariable Pendulums, made at the Melbourne Observatory.—June to September, 1893.*

By Pietro Baracchi, F.R.A.S.

[Read 13th October, 1893.]

The observations which form the subject of this paper were made with the three Invariable Pendulums of Kater's pattern marked 4, (1821) or 6, and 11, belonging to the Royal Society of London, and fully described in vol. v. of the account of *Operations of the Great Trigonometrical Survey of India*, and more recently by General Walker in the *Philosophical Transactions of the Royal Society*, vol. A., 1890, page 539. They were lent to Mr. Ellery for the purpose of being employed in the Gravity Survey of Australia proposed by the Royal Society of Victoria, and to be carried out by a committee specially appointed. These pendulums have been in existence over seventy years, and were swung in many parts of the world, at elevations ranging from sea-level to over 15,000 feet. As stated by General Walker, they were variously employed by Sabine, Bayley, Airy, and McClear, between the years 1822 and 1854. Captain Basevi took two of them, viz., No. 4 and No. 6 to India in 1864, where they were employed for eight years in gravity determinations, chiefly at stations along the Central Meridian Arc of the Great Trig. Survey.

In 1881 and 1882 Colonel Herschel swung them at Greenwich and Kew, together with Pendulum 11, and afterwards took the three to America, swinging them at Washington and Hoboken. After completing his operations they were given over to Mr. Edwin Smith of the U.S. Coast Survey, by whom they were taken round the world, and swung at Auckland, Sydney, Singapore, Tokio, San Francisco, and again at the starting point, Washington. Lastly, in 1888 and 1889, the Revisionary Operations with the three pendulums at Kew and Greenwich were undertaken, in consequence of discordance in the results of 1882.

They arrived in Melbourne in November, 1892, and Mr. Ellery made the necessary preparations for swinging them at the Observatory, and had all the various parts mounted and adjusted and ready for work by the middle of June last, when Mr. Love started the observations. At about the same time Lieut. Elblein of the Austrian warship *Saida*, swung three ½-second Pendulums of Colonel von Sterneck's type, at the Melbourne Observatory, thus connecting it with an independent basis, viz.: Vienna, where the absolute value of the Force of Gravity was determined by Professor Oppolzer. This increased the importance of the Melbourne swings with the Kater's Pendulums, for their results could be tested by two different series. It was decided in consequence, to extend the operations further than it was originally intended, taking for standard the revisionary work at Greenwich and Kew, and thus establish a satisfactory basis to which other Gravity determinations in Australia, made with Kater's or any different form of pendulums, might be referred. The Melbourne Observatory possessed all the necessary conditions for carrying out the observations under the best advantages.

The value of Gravity determinations by the differential method depends on the invariability of the pendulums used. There is evidence to show that when carefully handled, these pendulums remain unchanged and may be swung without re-grinding the knife edges for many years, and as the three Pendulums now employed have not been intentionally altered, or known to have met with any accident since Colonel Herschel swung them at Kew and Greenwich in 1882, it may be expected that they are now in the same condition as they were eleven years ago. Fortunately we have the means of testing this condition by swinging them again at Sydney, which it is to be hoped will be done, at the earliest opportunity.

The pendulums were swung in the eastern underground room in the main building of the Melbourne Observatory, which, having a stone pavement, offered all the required stability, and had also the advantage of keeping a fairly constant temperature. The clock by Shelton, the same as that used in England and India, was fixed to the south wall of the room, and the Pendulum Stand

and Observing Telescope were in a line with it, in the direction of the meridian. An image of the white disc attached to the pendulum of the clock was formed at the tail piece of the Invariable Pendulum, by the proper collimating lens, and was brought into the field of view of the Observing Telescope by a plain mirror, fixed at the object end, which could rotate in a plane at right angles to the meridian, thus enabling the observer to bring the eye end at any convenient altitude. The distances of the several parts from the tail piece were as follows :—Observing Telescope, o.g. 70·3 inches ; disc on Shelton pendulum, 48 inches ; collimating lens, 12·2 inches ; arc scale, about 1 inch for No. 11, and 0·5 inches for No. 4 and No. 6. The arc scale was about 18 inches, and the plane of suspension about 68 inches above the floor of the room, which is 84 feet above sea-level.

The operations consisted in observations of coincidences, temperature, pressure and arc of vibration, and comparison of the Shelton clock, with the sidereal standard clock of the Observatory, the rates of which are always known from transit observations, and finally, the reduction of observations.

Observation of Coincidences.

The diaphragm in front of the disc carried by the pendulum of the Shelton clock was so adjusted as to have its inner edges tangential to the disc, when at rest. (The disc is of white card two inches in diameter and turned to a true circle in the lathe). The image of the disc formed at the tail piece by the collimating lens was made of the same width as the tail piece, and the arc of vibration at the commencement was made a little less than the arc of the clock. The Disappearance *D*, and the Reappearance *R*, of the apparent right edge of the disc (actually the eastern edge) was invariably observed. The times of the first four swings were observed by the Seth Thomas, a sidereal clock kept in the room for the purposes of the Observatory, which, being near and almost in front of the observer, could be seen better than the Shelton, and had the advantage of being directly compared with the transit clock on the tape chronograph ; but this method introduced confusion, and was abandoned. For all the remaining swings, the times of D and R were observed by the Shelton; the D taking place between an even and odd second, and R between an odd and even second. If the size of the segment of the disc, when last seen before D, was larger than the segment when first seen after R, one second was added to the even second of time immediately preceding D, and if the opposite occurred, one second was subtracted from the odd second of time immediately following R. This refinement was not necessary; but as the method required only a very little more attention it was followed throughout.

Temperature.

The temperature of the pendulum was frequently observed during each swing, and was shown by the two thermometers Fahr. Nos. 667 and 668, placed on the dummy pendulum inside the cylinder. The adopted correction to the mean reading of the two, was $-0^{\circ}.13$.

During the first few swings the gas was allowed to burn in the room; but this brought about irregular changes of temperature, and caused the upper thermometer to read higher than the lower, often by as much as 0·4 degrees. The gaslight was accordingly discarded. For the rest of the time, till the conclusion of the observations, the temperature in the room did not vary by as much as one degree Fahr., and the two thermometers read always nearly alike.

Two Richard's thermographs, No. 1577 and No. 3131, were placed close to the cylinder, giving a continuous record of the temperature. During the first twelve swings the changes were so irregular and large, amounting sometimes to 3° or 4° Fahr., that curves had to be formed and intergrated in order to obtain a satisfactory mean; but for the remaining swings, the mean of the observed temperatures at commencement and at the end of the swing, as shown by the two thermometers on the dummy with the correction—0·13 applied, was adopted as representing with sufficient accuracy the mean temperature of the pendulum.

Pressure of Air.

The pendulums were swung at the ordinary atmospheric pressure, which was shown by the barometer Neuman, No. 122, having a correction of—0·022. The extreme range from June to the middle of September was from 29·4 to 30·4 inches. Pressure and temperature readings were always taken, at intervals of about one hour generally; sometimes more frequently.

Arc of Vibration.

This was read on the arc scale in inches. In the earlier part of the operations, the arc was read frequently, and several intermediate coincidences were observed during a swing; but later, the arc was read only twice at commencement, being generally about 0·65 inches, and twice at the end of the swing, being then reduced to about 0·08 inches.

Levelling.

The planes of suspension were carefully levelled before the pendulum was suspended, and again when the pendulum was taken out of the cylinder. No other intermediate observations of level were made; but the scale at right angles to the arc scale was read for each swing. It was found that for pendulum No. 4 and No. 6 the reading of this scale was about 0·5, and remained fairly constant in both positions of the marked face; but the readings for pendulum No. 11, were about 0·9, for face "P" and 1·2 inches for face "M". According to the level, the planes of the three pendulums generally remained in good adjustment, during each respective series of swings. Mr. Ellery invariably placed the pendulums in and out, and changed the position of the marked face when required.

The Swings.

Each pendulum was swung an equal number of times with its marked face towards the observer, and towards the clock, the first position being designated by face "M," and the second by face "P." Twelve sets were observed for each pendulum; six in each position. The results for Pendulum No. 11 came out more discordantly than those of the other two pendulums. It was feared that the ends of the knife edge sometimes touched one or other of the guiding faces close to the grooves which receive the knife edge when the pendulum is lifted off the planes. These pieces were taken off by Mr. Ellery on 6th September, and another series of twelve swings, six in each position, were observed under the new condition. This makes in all sixty swings of an average duration of five hours each. The general practice was to commence a swing at about 9.30 a.m., conclude it at about 3 p.m., commence another at 4 p.m., concluding at 9.30 p.m. Generally five coincidences were observed at the beginning and end of each swing. The pendulum was always started about half-an-hour before the first coincidence was observed.

Clock Rates.

Until the 12th August the Shelton Clock was compared with the Seth Thomas by eye and ear, the fraction of the second being determined by coincidences of beat, with a Mean Time Chronometer. The rates of the Seth Thomas were derived by chronographic comparison with the Standard Transit Clock, the error of which was determined by transit observations. After the 12th August, Mr. Ellery mounted an electric contact spring on the Shelton Clock, by which a signal was made at every sixtieth second, on the Tape Chronograph, on which the beats of the Transit Clock were simultaneously recorded, thus enabling a comparison of the two clocks to be made with all the accuracy obtainable. The uncertainty introduced by one of the weakest points in pendulum observations was, by this method of comparison, greatly reduced. A comparison was made at the commencement and end of each swing; but when two sets of swings were observed on the same day in succession, extending from 9 a.m. to 10 p.m., the rate of the Shelton Clock was derived from the two extreme comparisons in the morning and evening only, neglecting the two intermediate ones. In this way it was thought, although the rates for the two swings might be different, the resulting mean of the two vibration numbers would be improved. The Shelton Clock gave a good account of itself.

From 23rd June to 18th July it had a losing rate, gradually increasing from $1^s \cdot 9$ to $3^s \cdot 0$. It was then stopped to form the connection above mentioned, when the pendulum was slightly shortened. From 14th August to 14th September its meam gaining rate was $2^s \cdot 5$ per day, the greatest variation from mean being about $0^s \cdot 5$.

Reduction of the Observations.

The instructions contained in General Walker's *Memoranda on Pendulum Observations for the Melbourne Observatory* were followed throughout, excepting some slight variation in form, and in the co-efficient in the formulæ for the arc correction. The notation and formulæ used are as follows, viz:—If n is the number of observable coincidences during an interval I, and N the interval between two consecutive coincidences, then $I=(n-1)\cdot N$

I is obtained by subtracting the first three, four, or five from the last corresponding three, four, or five observed coincidences and taking the mean of the differences. N being approximately known from observations of two consecutive coincidences n is at once derived, with which the mean value of N is computed.

R = number of sidereal seconds in a mean solar day plus the daily rate of the clock Shelton.

V_1 = uncorrected number of vibrations made by the free pendulum in a mean solar day, then $V_1 = R - \frac{2R}{N}$

The vibration numbers V_1 resulting from each swing, were all firstly reduced to the mean temperature 62° Fahr., to 26 inches pressure at 32° Fahr., and to infinitely small arc, by the following formulæ, viz.:—

$$\text{Pressure correction} = 0\cdot34.\frac{B-26}{1+0\cdot0023\,(T-32)} = \beta$$

$$\text{Arc correction} = V.\left(\frac{D-d}{16rD}\right)^2 \cdot \left\{ (a+b)^2 - \tfrac{1}{3}(a-b)^2 \right\} = a$$

$$\text{Temperature correction} = 0\cdot45(T-62^\circ) = \tau$$

In which,

B = Observed reading of Barometer – 0·022 and reduced to 32° Fahr.

T = Mean Temperature of the Pendulum during the swing.

a and b.—The observed Arc of Vibration in inches at the commencement, and at the end.

D, d, and r.—The distances of the Arc Scale from the observing telescope, tail piece and knife edge respectively. It was adopted for the three pendulums. $V.\left(\frac{D-d}{16r.D}\right)^2 = 0\cdot13$

Hence,

$$\text{Arc correction} = 0\cdot13\Big((a+b)^2 - \tfrac{1}{3}(a-b)^2\Big)$$

And if V = number of vibrations in a mean solar day reduced to 62° Fahr., 26 inches pressure and infinitely small arc, we have $V = R - \frac{2R}{N} + \alpha + \beta + \tau$. This is the formula used in the reductions.

The general results are given in the following tables: Note—A full account of these operations will appear in the publications of the Melbourne Observatory.

MELBOURNE OBSERVATORY.—Collected results of Pendulum Swings (June—September, 1893). Being the number of Vibrations made by each Pendulum in a Mean Solar Day. Reduced to Mean Temperature 62° Fahr., 26 inches Pressure at 32° Fahr., and infinitely small arc. Height above sea-level, 86·5 feet.

TABLE I.

PENDULUM No. 4.

FACE M. No. of Swing.	FACE M. Date.	FACE M. No. of Vibrations.	FACE P. No. of Swing.	FACE P. Date.	FACE P. No. of Vibrations.			
9	July - - 15	86099·39	11	July - - 17	86099·24	Face M	...	86099·45
10	,, - - 17	86099·41	12	,, - - 18	86099·22	Face P	...	86099·09
15	August - 16	86099·55	13	August - 14	86099·07	Mean	...	86099·27
16	,, - 17	86099·44	14	,, - 15	86099·26			
17	,, - 17	86099·47	20	,, - 19	86098·96			
18	,, - 18	86099·31	21	,, - 20	86099·04			
19	,, - 18	86099·48	22	,, - 21	86098·92			
24	,, - 22	86099·54	23	,, - 21	86099·05			

PENDULUM No. 6.

FACE M. No. of Swing.	FACE M. Date.	FACE M. No. of Vibrations.	FACE P. No. of Swing.	FACE P. Date.	FACE P. No. of Vibrations.			
5	July - - 7	85999·25	7	July - - 9	85999·35	Face M	...	85999·43
6	,, - - 8	85999·63	8	,, - - 10	85999·27	Face P	...	85999·43
25	August - 25	85999·45	31	August - 28	85999·38	Mean	...	85999·43
26	,, - 26	85999·31	32	,, - 29	85999·25			
27	,, - 26	85999·45	33	,, - 29	85999·30			
28	,, - 27	85999·59	34	,, - 30	85999·68			
29	,, - 27	85999·48	35	,, - 30	85999·70			
30	,, - 28	85999·29	36	,, - 31	85999·46			

PENDULUM No. 11.

FACE M.			FACE P.		
No. of Swing.	Date.	No. of Vibrations.	No. of Swing.	Date.	No. of Vibrations.
3	June - - 25	86050·98	1	June - - 23	86050·10
4	,, - - 26	86050·36	2	,, - - 24	86050·56
37	August - 31	86051·40	43	September 3	86050·07
38	September 1	86051·30	44	,, 4	86050·30
39	,, 1	86051·49	45	,, 4	86050·73
40	,, 2	86050·93	46	,, 5	86050·99
41	,, 2	86051·08	47	,, 5	86051·10
42	,, 3	86051·10	48	,, 6	86051·07
55	September 10	86051·13	49	September 6	86051·55
56	,, 11	86051·08	50	,, 7	86051·74
57	,, 11	86051·22	51	,, 7	86051·62
58	,, 12	86050·97	52	,, 8	86051·48
59	,, 12	86050·97	53	,, 9	86051·55
60	,, 13	86051·08	54	,, 10	86051·74

Face M	..	86051·08
Face P	...	86051·04
Mean	...	86051·06

In order to make the vibration numbers just given comparable with the Greenwich and Kew results, they must be reduced to vacuum and sea-level. For the reduction to vacuum, using the same formula as before, we have :

$$\text{Correction} = 0{\cdot}34 . \frac{26}{1 + 0{\cdot}0023 \times 30} = 8{\cdot}269$$

and treating the correction for height of station, in the same way, as General Walker did for Kew and Greenwich (see *Phil. Trans.*, v. 1890, A., page 557), viz.—Correction $= \frac{h}{243}$ in which the height h may be put as 86·5 feet, we have

$$\text{Correction to sea-level} = \frac{86{\cdot}5}{243} = 0{\cdot}355$$

$$\text{Total correction} = +8{\cdot}62$$

The following table shows the concluded vibration number for each pendulum at the Melbourne Observatory.

TABLE II.

No. of Pendulum.	Number of Vibrations in a Mean Solar Day, reduced to the Mean Temperature 62° Fahr., infinitely small Arc, and		
	To Pressure of Air 26 inches at 32° Fahr.	To a Vacuum.	To a Vacuum and Sea-level.
4	86099·27	86107·54	86107·89
6	85999·43	86007·70	86008·05
11	86051·06	86059·33	86059·68

COMPARISON OF THE VIBRATION NUMBERS OF THE THREE INVARIABLE PENDULUMS AT GREENWICH, KEW, SYDNEY AND MELBOURNE.

As no swings under low pressure have been observed at Melbourne, only the results of swings under high pressure observed at Greenwich and Kew are taken into account in this comparison.

The following numbers are taken from General Walker's paper in the *Phil. Trans.*, vol. 1890 A., page 551, 553 and 558.

TABLE III.

Number of Pendulum.	Number of Vibrations in a Mean Solar Day reduced to Mean Temperature 62° Fahr., infinitely small Arc and to a Vacuum.		Number of Vibrations in a Mean Solar Day reduced to Mean Temperature 62° Fahr., infinitely small Arc and to the Density of Air under the Pressure of 26 inches at 32° Fahr.
	Greenwich Results, 1889, by Mr. Hollis.	Kew Results, 1888, by Mr. Constable.	Sydney Results, 1883, by Mr. Edwin Smith.
4	86164·04	86165·27	86090·93
6	86063·94	86065·80	85990·32
11	86115·68	86116·04	86042·08

General Walker states in his paper above cited that the Greenwich and Kew swings were reduced to a vacuum by using the Kew formula,

$$\text{Correction} = 0{\cdot}32\frac{\beta}{1 + {\cdot}0023(\tau - 32)}$$

Hence in order to make them comparable with the Melbourne swings, they must be increased by the quantity

$$\left(0{\cdot}34 - 0{\cdot}32\right).\frac{27}{1 + {\cdot}0023 \times 30} = +0{\cdot}505.$$

They must also be reduced to sea-level by applying the corrections $\frac{157}{243} = +0{\cdot}646$ for Greenwich and $\frac{15}{243} = +0{\cdot}061$ for Kew.

For Sydney the reduction to sea-level is $\frac{140}{243} = +0{\cdot}576$

$$\text{Reduction to a vacuum } 0.34.\frac{26}{1 + 0{\cdot}0023 \times 30} = +8{\cdot}269$$

The whole correction to be applied to the numbers in Table III. is therefore :

To Greenwich	numbers	+ 1·15
„ Kew	„	+ 0·57
„ Sydney	„	+ 8.85

TABLE IV.

Number of Pendulum.	Number of Vibrations made in a Mean Solar Day, reduced to a Vacuum, to Temperature 62° Fahr., to infinitely small Arc and to Sea-level.				Differences.				
	Greenwich.	Kew.	Sydney.	Melbourne.	G-M.	K-M.	M-S.	K-G.	K-S.
4	86165·19	86165·84	86099·78	86107·89	+ 57·30	+ 57·95	+ 8·11	+ 0·65	+ 66·06
6	86065·09	86066·37	85999·17	86008·05	+ 57·04	+ 58·32	+ 8·88	+ 1·28	+ 67·20
11	86116·83	86116·61	86050·93	86059·68	+ 57·15	+ 56·93	+ 8·75	− 0·22	+ 65·68
	Mean Differences - - - -				+ 57·16	+ 57·73	+ 8·58	+ 0·57	+ 66·31

This table (No. IV.) embodies the conclusion of the differential results. It will be seen that by diminishing the vibration number of Pendulum No. 6, and by increasing the vibration number of Pendulum No. 11 by (say) 0·5 vibrations in the column for Kew, the columns containing the differences would be brought into much closer adjustment. If comparisons of several clocks are involved in the operations, errors of 0·5 vibrations could be very easily introduced.

For the conversion of the differential results into absolute measure of the force of Gravity for Melbourne, we have now the Greenwich, Kew, and Vienna bases, and Professor Neumayer's independent value of g.

The absolute length of the seconds pendulum at Greenwich, was determined by General Sabine in 1830, by the convertible pendulum originally designed and employed by Captain Kater in 1817, and found to be 39·13734 inches. (See *Phil. Trans.*, 1831, Art. xxv).

In 1873 Major Heaviside determined the length of the pendulum vibrating seconds at Kew, using the same pendulum as that used by General Sabine at Greenwich; its length was remeasured by Colonel Clarke. As the result of these observations, the length of the seconds pendulum at Kew was determined to be 39·14008 inches, and this result was corroborated by the operations with the two reversible Russian pendulums at Kew, but these pendulums do not seem to have given satisfactory results elsewhere (as shown in the volume v. of *The Great Trigonometrical Survey of India).*

Professor Neumayer determined the value of g at Melbourne in 1863, by a reversible pendulum constructed by Mr. Lohmeir of Hamburg, under the supervision of Professor Peters of the Altona Observatory. Of these operations no detailed account seems to be available, and it would be desirable to know something more about them, so as to form a judgment as to the weight to be given to the result, before comparing it with the others.

Lieutenant Elblein gave to Mr. Ellery the following provisional results of his $\frac{1}{2}$ seconds invariable pendulum observations at Melbourne and Sydney:

Period of one vibration, reduced to 0° centigrade, to infinitely small arc, and to a vacuum, being the mean of three ½ seconds pendulums

At Melbourne (86·5 feet above sea-level) $0^s \cdot 5066120$
At Sydney (140 feet above sea-level) $0^s \cdot 5063920$

and taking for the value of g at Vienna $g = 9{\cdot}80866$ meters as found by Professor Oppolzer by a reversible pendulum in the year 1886, he derived the following values, not reduced to sea-level.

Melbourne $g = 9{\cdot}80014$ meters
Sydney $g = 9{\cdot}79702$ meters

And reduced to sea-level,

Melbourne $g = 9{\cdot}80020$ meters
Sydney $g = 9{\cdot}79713$ meters

According to these values, the Indian pendulums should make 13·48 vibrations less at Sydney than the number of vibrations they make at Melbourne in a mean solar day. Therefore the difference 8·58 given in Table IV. is too small by nearly 5 vibrations, according to Lieutenant Elblein's provisional results.

This fact casts a doubt either on some of the observations or on the invariability of the pendulums, as the discordance is quite independent of the absolute values chosen as bases, and small differences in the formulæ used for computing the various corrections could not account for such a large error. It is therefore all the more urgent to swing the Pendulums at Sydney at the very first opportunity.

The several values of g for Melbourne derived from the above sources are as follows, viz.:—

By the Greenwich and Melbourne swings, and the length of the seconds pendulum as 39·13734 inches at the former place	$g = 32{\cdot}14645$ feet $g = 9{\cdot}79815$ meters
By the Kew and Melbourne swings, and the length of the seconds pendulum as 39·14008 at the former place	$g = 32{\cdot}14827$ feet $g = 9{\cdot}79870$ meters
Provisional value of Lieutenant Elblein	$g = 32{\cdot}15317$ feet $= 9{\cdot}80020$ meters

Professor Neumayer's absolute determination $\left.\right\} g = 32{\cdot}15127$ feet $= 9{\cdot}799607$ meters

There is no urgent need for adopting at once a final value for g—at least not until the pendulums are swung again in England on their return.

The mean of the 4 values is 32·14979 feet or 9·79916 meters, which may be provisionally adopted.